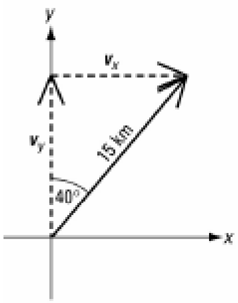
Adding Vectors Redux (Key)

#1

$\Delta\vec{d} = 15$ km [40° E of N]



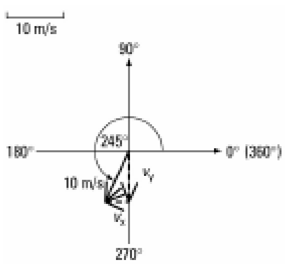
The angle is given with respect to the y-axis (E of N), so use the cosine function to calculate the north component:

$\Delta d_y = (15 \text{ km})(\cos 40°)$

$= 11$ km [N]

#2

$\vec{v} = 10$ m/s [245°]



$v_x = (10 \text{ m/s})(\cos 245°)$

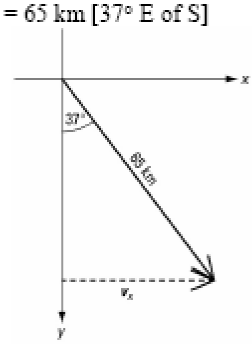
$= -4.2$ m/s

$v_y = (10 \text{ m/s})(\sin 245°)$

$= -9.1$ m/s

#3

$\Delta\vec{d} = 65$ km [37° E of S]



$\Delta d_x = (65 \text{ km})(\sin 37°) = 39$ km [E]

#4

. ***Given***

$\vec{d}_1 = 80.0 \text{ m } [0°]$

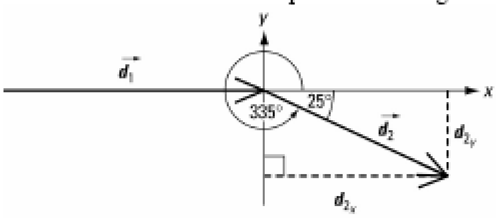
$\vec{d}_2 = 60.0 \text{ m } [335°]$

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve both vectors in their components and then perform vector addition.
Determine the resultant displacement using the Pythagorean theorem.



$$d_{1_x} = 80.0 \text{ m}$$

$$d_{1_y} = 0$$

$$\begin{aligned} d_{2_x} &= d_2 \cos\theta \\ &= (60.0 \text{ m})(\cos 25°) \\ &= 54.38 \text{ m} \end{aligned}$$

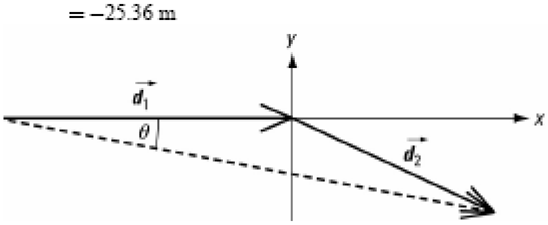
$$\begin{aligned} d_{2_y} &= d_2 \sin\theta \\ &= -(60.0 \text{ m})(\sin 25°) \\ &= -25.36 \text{ m} \end{aligned}$$

$$\Delta d_x = d_{1_x} + d_{2_x}$$

$$\begin{aligned} \Delta d_x &= 80.0 \text{ m} + 54.38 \text{ m} \\ &= 134.4 \text{ m} \end{aligned}$$

$$\Delta d_y = d_{1_y} + d_{2_y}$$

$$\begin{aligned} \Delta d_y &= 0 + (-25.36 \text{ m}) \\ &= -25.36 \text{ m} \end{aligned}$$



$$\begin{aligned} \Delta d &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2} \\ &= \sqrt{(134.4 \text{ m})^2 + (-25.36 \text{ m})^2} \\ &= 137 \text{ m} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{25.36 \text{ m}}{134.4 \text{ m}} \\ &= 0.1887 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}(0.1887) \\ &= 10.69° \end{aligned}$$

Since the angle is with respect to the positive x-axis in the negative direction, the angle with respect to the positive x-axis in the positive direction is $360° - 10.69° = 349°$.

Paraphrase

The farmer's displacement was 137 m [349°].

#5

Given

$\vec{d}_1 = 1.20 \text{ km [55° N of E]}$

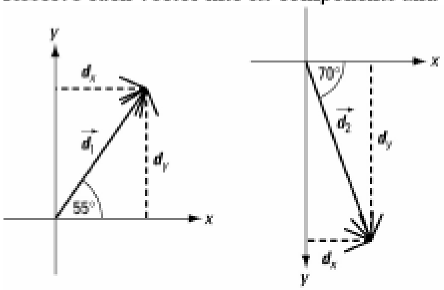
$\vec{d}_2 = 3.15 \text{ km [70° S of E]}$

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Resolve each vector into its components and perform vector addition.



$$\begin{aligned} d_{1_x} &= d_1 \cos\theta \\ &= (1.20 \text{ km})(\cos 55°) \\ &= 0.6883 \text{ km} \end{aligned}$$

$$\begin{aligned} d_{1_y} &= d_1 \sin\theta \\ &= (1.20 \text{ m})(\sin 55°) \\ &= 0.9830 \text{ km} \end{aligned}$$

$$\begin{aligned} d_{2_x} &= d_2 \sin\theta \\ &= (3.15 \text{ km})(\sin 20°) \\ &= 1.077 \text{ km} \end{aligned}$$

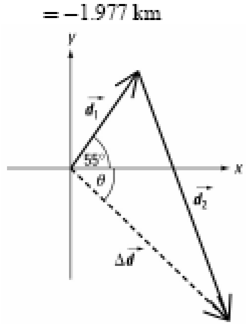
$$\begin{aligned} d_{2_y} &= -d_2 \cos\theta \\ &= -(3.15 \text{ km})(\cos 20°) \\ &= -2.960 \text{ km} \end{aligned}$$

$$\Delta d_x = d_{1_x} + d_{2_x}$$

$$\begin{aligned} \Delta d_x &= 0.6883 \text{ km} + 1.077 \text{ km} \\ &= 1.766 \text{ km} \end{aligned}$$

$$\Delta d_y = d_{1_y} + d_{2_y}$$

$$\begin{aligned} \Delta d_y &= 0.9830 \text{ km} - 2.960 \text{ km} \\ &= -1.977 \text{ km} \end{aligned}$$



$$\begin{aligned} \Delta d &= \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2} \\ &= \sqrt{(1.766 \text{ km})^2 + (-1.977 \text{ km})^2} \\ &= 2.651 \text{ km} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{\text{opposite}}{\text{adjacent}} \\ &= \frac{1.977 \text{ km}}{1.766 \text{ km}} \\ &= 1.120 \end{aligned}$$

$$\begin{aligned} \theta &= \tan^{-1}(1.120) \\ &= 48° \end{aligned}$$

From the diagram, this angle is with respect to the positive *x*-axis in the clockwise direction, or S of E.

Paraphrase

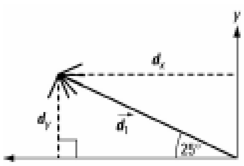
The jet ski's displacement is 2.65 km [48° S of E].

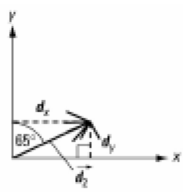
#6

Given
$\vec{v}_1 = 6.0$ km/h [25° N of W]
$\vec{v}_2 = 4.5$ km/h [65° E of N]
$\Delta t_1 = 35$ min
$\Delta t_2 = 20$ min
Required
displacement ($\Delta \vec{d}$)
average velocity ($\vec{v}_{ave}$)
Analysis and Solution
Using the uniform motion equation, determine the displacement in both directions.

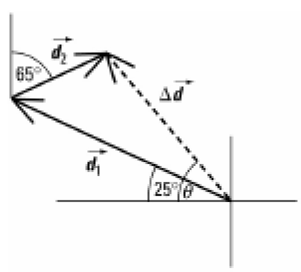
$$\Delta d_1 = \left(6.0\ \frac{\text{km}}{\cancel{\text{h}}}\ [25° \text{ N of W}]\right)\left(35\ \cancel{\text{min}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)$$
$$= 3.5 \text{ km } [25° \text{ N of W}]$$
$$\Delta d_2 = \left(4.5\ \frac{\text{km}}{\cancel{\text{h}}}\ [65° \text{ E of N}]\right)\left(20\ \cancel{\text{min}} \times \frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right)$$
$$= 1.5 \text{ km } [65° \text{ E of N}]$$

Resolve each displacement vector into its x and y components and then perform vector addition.





$$\Delta d_{1_x} = -d_1 \cos\theta$$
$$= -(3.5 \text{ km})(\cos 25°)$$
$$= -3.17 \text{ km}$$
$$d_{1_y} = d_1 \sin\theta$$
$$= (3.5 \text{ m})(\sin 25°)$$
$$= 1.48 \text{ km}$$
$$d_{2_x} = d_2 \sin\theta$$
$$= (1.5 \text{ km})(\sin 65°)$$
$$= 1.36 \text{ km}$$
$$d_{2_y} = d_2 \cos\theta$$
$$= (1.5 \text{ km})(\cos 65°)$$
$$= 0.634 \text{ km}$$
$$\Delta d_x = d_{1_x} + d_{2_x}$$
$$= -3.17 \text{ km} + 1.36 \text{ km}$$
$$= -1.81 \text{ km}$$
$$\Delta d_y = d_{1_y} + d_{2_y}$$
$$= 1.48 \text{ km} + 0.634 \text{ km}$$
$$= 2.11 \text{ km}$$



$$\Delta d = \sqrt{(\Delta d_x)^2 + (\Delta d_y)^2}$$
$$= \sqrt{(-1.81 \text{ km})^2 + (2.11 \text{ km})^2}$$
$$= 2.78 \text{ km}$$
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$
$$= \frac{2.11 \text{ km}}{1.81 \text{ km}}$$
$$= 1.17$$
$$\theta = \tan^{-1}(1.17)$$
$$= 49°$$

From the diagram above, this angle is 49° N of W.

Find average velocity using the equation $\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$.

$$\vec{v}_{ave} = \frac{\Delta\vec{d}}{\Delta t}$$
$$= \frac{2.78 \text{ km } [49° \text{ N of W}]}{(35+20)\ \cancel{\text{min}}} \times \frac{60\ \cancel{\text{min}}}{1 \text{ h}}$$
$$= 3.0 \text{ km/h } [49° \text{ N of W}]$$

Paraphrase
The jogger's total displacement is 2.8 km [49° N of W]. The jogger's average velocity is 3.0 km/h [49° N of W].